# PARALLEL 3D FINITE ELEMENT ANALYSIS OF NUCLEAR REACTOR PRESSURE VESSELS

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## Abstract

Debate regarding climate change has led to a renewed interest in nuclear energy generation. Safety is still the key engineering issue and various groups worldwide are studying the structural behaviour of nuclear reactor vessels. For example, a recent project undertaken by a 9-partner European consortium (MAECENAS 2001-2004) sought to model complex coupled processes occurring over the life-time of a reactor. The need to represent three spatial dimensions, multi-physics and time dependency leads to significant computational demands. In this paper, the authors discuss the performance characteristics of two parallelisation strategies that can potentially be used to help solve this type of problem: (i) a parallel direct multiple frontal matrix solver and (ii) a parallel element-by-element preconditioned conjugate gradient solver. The study shows that a full 3D static linear elastic analysis of the MAECENAS pressure vessel model, using a typical supercomputer, was only possible using the latter technique.

**Keywords:** Parallel computing, element-by-element, finite elements, nuclear engineering.

## Introduction

In recent work, the authors have considered two scenarios: (i) creep and continuum damage modelling in welds over the lifetime of a steel nuclear reactor pressure vessel [1] and (ii) a fully coupled thermo-hygro-mechanical analysis of a pre-stressed concrete pressure vessel (the MAECENAS scenario) [2].

In the first case, non-linear creep and damage mechanics can be modelled as a sequence of approximately linear steps, with a linear solver being used to solve the system of equilibrium equations at each time step. Recent work indicates that around 10,000 steps would be required to predict structural behaviour over a 20-30 year lifetime [3], giving an indication of the significant computational demands for a relatively simple process.

The second case requires constitutive models to describe the interaction of the different physical processes. These include the effects of heat, pressure and chemical reactions: all

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exacerbated by the presence of water. The relationship describing the interaction between these processes is non-linear and an incremental technique, with approximately linear sub-steps could also be used to model the system.

The point being made is that at the heart of these finite element simulations are repeated linear solves. Assuming that most of the time is spent in the solver, then using a parallel one promises to greatly reduce solution times and enable fully 3D modelling.

## Comparison of Two Parallel Solvers

A recent study has been carried out [1] that compares the performance characteristics of a parallel direct multiple frontal matrix solver [4, 5] and a parallel element-by-element implementation of the preconditioned conjugate gradient (EBE PCG) method [6, 7]. The PCG solver is provided free in Manchester's ParaFEM library, with source code and example finite element programs (see www.parafem.org). The study was carried out using CSAR's 512 processor SGI Altix, hosted by the University of Manchester. The Altix processors are 1.3 Ghz Intel Itanium chips, each having access to 2 Gigabytes of local memory.

PCG is an iterative solver and the iterative process is terminated by a convergence test. In this study a convergence tolerance of 1.E-10 was chosen (maximum difference between current and previous answer). This was considered to be accurate enough to justify comparing solution times with the direct solver.

Table 1 compares solution time versus number of processors for the largest problem that could be solved using the direct solver: a tetrahedral mesh comprising 162,601 nodes or around half a million degrees of freedom. The next largest mesh attempted had 279,121 nodes and could not be run using the direct solver. This was because there was insufficient core memory available on the machine used. In contrast the PCG solver could be used to solve much larger problems as it is more memory efficient: for a fixed problem size, the memory required per processor decreases as the number of processors is increased.

| No of Processors | Iterative Solver Solution Time (s) | Iterative Speed-up | Relative Speed-up Iterative/Direct |
|---|---|---|---|
| 1 | 1797 | 1 | - |
| 2 | 941 | 1.91 | 6.45 |
| 4 | 476 | 3.77 | 8.38 |
| 8 | 245 | 7.34 | 15.30 |
| 16 | 125 | 14.33 | 45.48 |
| 32 | 67 | 26.98 | - |
| 64 | 37 | 48.69 | - |

**Table 1 Performance Comparison between Iterative and Direct Solvers**

The direct solver does not run on a single processor and a memory fault was generated when more than 16 processors were used. The final column of the table compares the iterative and direct solution times. For example, using 16 processors, the parallel iterative solver was 45 times faster than the parallel direct solver. The direct solver solution times include factorisation of the global stiffness matrix, a process that is notoriously costly (in both time and memory) and not required in an iterative solver.

## The MAECENAS Model

During the European funded MAECENAS project (2001-2004), a 9-partner European consortium worked with a common finite element model of a pre-stressed concrete nuclear reactor pressure vessel [2]. The model, shown in Figure 1, comprises 108,788 20-node hexahedral elements and 473,094 nodes. Considering displacement degrees of freedom only, the model is represented by 1,419,282 equations.

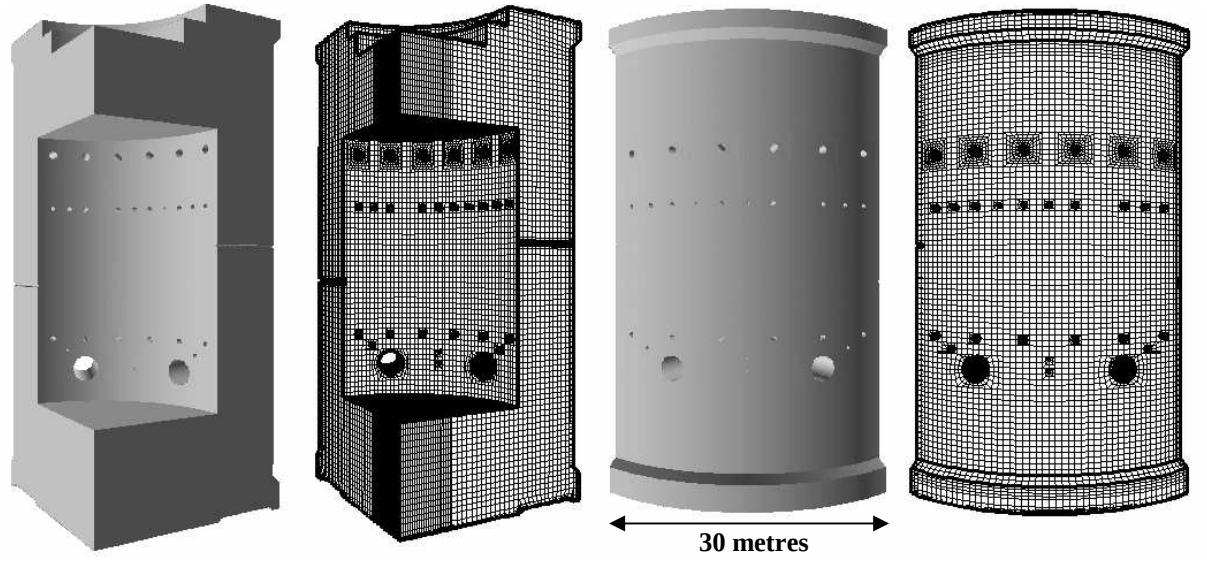


**Figure 1 Quarter section of the pressure vessel**

Even if one only wanted to study the linear elastic deformation of the pressure vessel, the previous section shows that this would not be possible at all using the direct solver. Of course, the core memory made available on the machine could be increased using 64-bit addressing. However, the EBE PCG solver is more memory efficient and larger problems simply require a larger machine (more processors).

A linear elastic analysis has been carried out in which the response of the pressure vessel to a uniform internal pressure has been determined. A time-dependent or non-linear analysis could comprise many such approximately linear steps or increments. Table 1 shows the total solution time (for a tolerance of 0.001) versus number of processors used. Times shown are total wall-clock times and include data input, data partitioning and distribution, solving the equations, collecting the results and output. These are the times the engineer would actually wait to receive the results. If serial operations such as I/O and data distribution were not included, the speed-up figures would be significantly better – representing the true scalability of the computational kernel, the EBE PCG solver. This is important when considering thousands of repeated solves typical of non-linear or time dependent modelling. No specific optimisations have been carried out and the authors note that there are further potential improvements to be made, particularly in "domain composition". Using the EBE method, there is no global matrix and subdomains are composed, rather than decomposed.

| Processors | 1 | 2 | 4 | 8 | 16 | 32 | 64 | 128 |
|---|---|---|---|---|---|---|---|---|
| Time | 1870 | 1055 | 530 | 274 | 147 | 83 | 55 | 53 |
| Speed-up | 1 | 1.77 | 3.53 | 6.82 | 12.72 | 22.53 | 34.00 | 35.28 |

**Table 2 Solution times for the MAECENAS Model**

Finite element analysis results are presented in Figure 2 below. In Figures 2a, 2b and 2c, a greyscale colour map has been used to represent mean displacement. The range, dark to light, represents small to large displacements. Figure 2a shows the deformed geometry. A "zebra" colour map is used in Figure 2d, greyscale with overlying dark lines joining points of equal value. In contrast to the other images, Figure 2d highlights the slight asymmetry of the deformation. The asymmetry is due to the features marked "x" in Figure 2c.

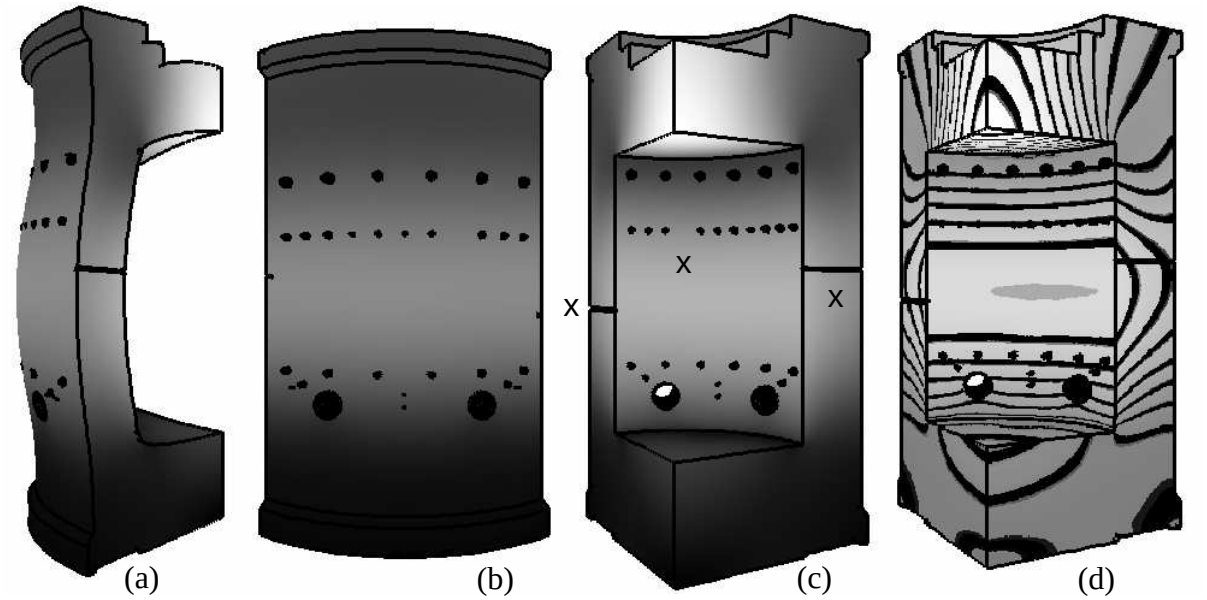


**Figure 2 Mean displacements under uniform internal pressure**

## Conclusions

The performance characteristics of two parallel solvers have been compared: (i) a direct multi-frontal matrix solver and (ii) an element-by-element preconditioned conjugate gradient solver. The EBE PCG solver gives significant reductions in solution time compared with the multi-frontal solver. The EBE PCG solver has been used to examine the structural response of the MAECENAS model for a static elastic load. On the same machine, this was not possible at all using the direct solver. For simulations that require many approximately linear steps to represent either non-linear behaviour or time dependency, use of the parallel EBE PCG solver brings distinct advantages with respect to both size of problem that can be solved and reductions in solution times. The authors note that the use of ParaFEM for solving dynamic and non-linear problems has been demonstrated elsewhere.

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